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by

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Abstract

In the orbit determination procedure described in this paper, the set of physical quantities fitted in the computations is not the set of basic measurements made by the ground stations. Rather, it uses the basic measurements on a pass-by-pass basis, together with a preliminary estimate of the orbit, to determine a fictitious position of the tracking station for each pass. It can be shown that the error in station position is also a representation of the error in satellite position at the center time of the pass. Thus, for each pass, a new estimate of satellite position is formed, and a new orbit is chosen to fit these positions. It should be possible to apply this method to any type of tracking data. With Doppler data, the convergence of the orbit determination position is much faster than that of the conventional method.

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RAPID DETERMINATION OF SATELLITE ORBITS FROM DOPPLER DATA

Introduction

The basic method used for determining satellite orbits from Doppler data is that of making a least squares fit to the Doppler data [1][1]. That is, given a set of measurements of Doppler frequency versus time, and a preliminary estimate of the orbit parameters, a theoretical calculation is made of the Doppler frequency at the observation times, based upon the preliminary parameters, and the parameters are then varied until the theoretical values best fit the observed values in a least-squares sense. In the accurate application of this basic method, there have always been two basic difficulties.

The first basic difficulty is that of eliminating spurious data, that is, data obtained when the tracking station is not operating accurately, or is not locked on to the satellite signal. The most reliable method found yet for doing this is that of the "navigation pre-processor" [2]. In this pre-processor, a preliminary orbit is combined with Doppler data for a single pass to "navigate" the station, that is, to find a position for the station that best fits the data, again in a least squares sense, assuming the given orbit to be correct. The residuals

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[1] Numbers in square brackets refer to the references.

for all individual data points are then calculated, and any point with a large residual is almost sure to be spurious[2].

The second difficulty is the slow convergence of the iterative process used in replacing the preliminary set of parameters by an improved set. The final orbits obtained are quite accurate [3], but many iterations are needed to find them. Typically, ten iterations are required to improve an orbit with an initial error of 2 km. r.m.s. to one that is at the accuracy limit allowed by the current knowledge of the gravity field, about $\frac{1}{4}$ km. r.m.s. This slow convergence may be tolerable when the data are being used for research, but is a serious problem in the day-to-day maintenance of an accurate orbit, and is an acute problem in the first orbit determination of a newly launched satellite. For example, the design orbit of 1962 $\beta\mu 1$ (ANNA 1B) differed from the actual orbit by only 1 part in 200 for the period, by .007 for the eccentricity, and by comparably small amounts for the other parameters, but about fifty iterations were required to find an orbit at the limiting level of accuracy.

This paper will show that the navigation pre-processor, which provided a superb answer to the data deletion problem, can also be used to yield a rapidly convergent orbit determination process. It is believed that this method has the same limiting accuracy as the standard method.

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[2] It is not the purpose of this paper to analyze data deletion, but one point should be noted. If the data points that are deleted depend upon the orbit used, the deletion is probably not valid. In the actual use of the method, the criteria for deletion have been chosen to make the set of rejected points highly stable with respect to variations in the orbit; thus the rejected data are not consistent with any possible orbit and are hence spurious.

General Description of the New Method

Guier [2] has used the position results given by the pre-processor to study the internal consistency of orbits determined from Doppler data. He bases this use upon a theorem that the error in station coordinates deduced for any pass is a valid representation of the error in satellite coordinates at the center of the pass, provided that the velocity error is negligible (as would be expected for an orbit determined by fitting data over a long time span).

This theorem obviously does not apply when using an orbit that is not yet well determined, and *a forteriori* does not apply in the initial orbit determination for a new satellite. However, let us assume for the moment that the inferred station position does represent the orbit position error with some adequacy even under these circumstances. We can then outline the following procedure for orbit determination.

Start with Doppler data for several passes (preferably from more than one station and spanning at least one orbital period in time), and with any estimate of the orbit, such as the design orbit for a new satellite or an orbit updated from an earlier epoch for an already tracked satellite. Next, take the data for each pass in turn, and determine a "position" for the station based upon the pass data and the orbit estimate (at the same time performing the deletion of spurious data), and determine the corresponding vector error in station position. Then calculate the position vector of the satellite in the assumed orbit at the center time

of the pass, and subtract the vector error in station position. Thus, for the center time of each pass, we have a new estimate of satellite position, and from this set of estimated positions, we can derive a new estimate of the orbit.

This method has one great advantage over the conventional method. It is necessary to handle the Doppler data only on a pass-by-pass basis and not over the entire tracking interval. Further, when the data are handled they are used only in estimating a station position, which has only three degrees of freedom; this involves less manipulation of the data than does the estimating of an orbit, with six degrees of freedom. In the [redacted] system, which involves thousands of data points[3] per day per satellite, this is an important advantage.

It is not a priori obvious whether the proposed method should have slower or faster convergence than the conventional method. In the few tests that have been performed to date, the convergence has been much faster, for reasons that are not yet understood.

It is now necessary to determine the effects that errors in the velocity vector of a satellite have upon the station coordinates inferred from that pass, in order to justify the basic assumption of the method.

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[3]This method should be superior to methods that involve compressing the data by fitting spans of data to arbitrary functions such as polynomials, and then computing synthetic data points from these functions. It is difficult to use arbitrary functions with an accuracy comparable to that of the basic data; with the current quality of Doppler data, it is necessary that the arbitrary functions be free of bias to about one part in 10^5 of the Doppler frequency, or about one part in 10^{10} of the received frequency. Using this viewpoint, one may describe the present method as one in which the Doppler data are replaced by three numbers, capable of representing all of the data for an entire pass without bias.

Effect of Satellite Velocity Error Upon Inference of Station Position

Let $\underline{r}_s$ be the correct position vector of an observing station, let $\underline{r}_o(t)$ be the correct orbit of a satellite, let

$$\underline{R} = \underline{r}_o - \underline{r}_s \ , \tag{1}$$

and let R by the magnitude of $\underline{R}$. The correct range rate w(t) is then

$$w(t) = (\dot{\underline{r}}_o \cdot \underline{R})/R \ , \tag{2}$$

neglecting the velocity of the station produced be the Earth's rotation. As long as the error in the estimated orbit dominates instrumental errors, we may also take this to be the experimental range rate measured by the Doppler shift.

Let $\underline{r}(t)$ be an approximate orbit and let

$$\delta\underline{r}(t) = \underline{r}(t) - \underline{r}_o(t) \tag{3}$$

be the error in this approximate orbit. Further, let $\underline{r}_n$ be an inferred station position, calculated by combining the experimental values of the range rate with the approximate orbit, and let

$$\underline{\rho} = \underline{r}_n - \underline{r}_s \tag{4}$$

be the error in inferred station position.

Using a station position $\underline{r}_n$ and an orbit $\underline{r}(t)$, the theoretically computed range rate $w_{th}(t)$ is

$$w_{th}(t) = [\dot{\underline{r}} \cdot (\underline{r} - \underline{r}_n)]/|\underline{r} - \underline{r}_n| \ .$$

If we eliminate $\underline{r}_n$ and $\underline{r}$ using Eqs. (3) and (4), and keep terms through first order in $\delta\underline{r}$ and $\underline{\rho}$, this can be written

$$w_{th}(t) = R^{-1}\{\dot{\underline{r}}_o \cdot \underline{R} + \dot{\underline{r}}_o \cdot (\delta\underline{r} - \underline{\rho}) + \underline{R} \cdot \delta\dot{\underline{r}} - (\dot{\underline{r}}_o \cdot \underline{R})[\underline{R} \cdot (\delta\underline{r} - \underline{\rho})/R^2]\} \ ,$$

in which $\underline{R}$ and R still refer to the correct orbit and station position, as defined in Eq. (1).

The residual $\delta w(t) = w_{th}(t) - w(t)$ at time t is hence

$$\delta w(t) = R^{-1}\{(\delta\underline{r} - \underline{\rho}).[\dot{\underline{r}}_o - \underline{R}((\dot{\underline{r}}_o \cdot \underline{R})/R^2)] + \underline{R} \cdot \delta\dot{\underline{r}}\} \ . \qquad (5)$$

The position error $\underline{\rho}$ is determined by varying $\underline{\rho}$ (but not $\delta\underline{r}$ or $\delta\dot{\underline{r}}$) until the r.m.s. of the residuals at all of the time points during a pass is a minimum.

Before minimizing the residuals, let us simplify $\delta w(t)$ somewhat. Defining a vector $\underline{v}_\perp$ by

$$\underline{v}_\perp = \underline{R} \times (\dot{\underline{r}}_o \times \underline{R})/R^2 \ , \qquad (6)$$

we can rewrite Eq. (5) as

$$\delta w(t) = R^{-1}\{\underline{v}_\perp \cdot (\delta \underline{r} - \underline{\rho}) + \underline{R} \cdot \delta \dot{\underline{r}}\} \tag{7}$$

$\underline{v}_\perp$ is a vector in the plane containing $\underline{R}$ and $\dot{\underline{r}}_o$, normal to $\underline{R}$, whose magnitude equals the component of satellite velocity normal to $\underline{R}$. We can further simplify by choosing the time origin. Since the time of closest approach can be determined with considerable accuracy even with a wrong orbit, take a specific epoch near this time as the origin. Then, if $\delta \underline{r}_i$ and $\delta \dot{\underline{r}}_i$ denote the position and velocity errors at this epoch, we can approximate $\delta \underline{r}(t)$ by $\delta \underline{r}_i + t\, \delta \dot{\underline{r}}_i$ during a pass. Then

$$\delta w(t) = R^{-1}\{\underline{v}_\perp \cdot (\delta \underline{r}_i - \underline{\rho}) + (t\underline{v}_\perp + \underline{R}) \cdot \delta \dot{\underline{r}}_i\} \quad . \tag{8}$$

Clearly, if $\delta \dot{\underline{r}}_i$ is negligible, the r.m.s. residual will be a minimum when $\underline{\rho} = \delta \underline{r}_i$, as Guier [2] stated.

To find $\underline{\rho}$, we square $\delta w(t)$, sum over all time points, and divide by the number of time points, thus forming the variance of the residuals. We then equate to zero the partial derivatives of the variance with respect to the components of $\underline{\rho}$, and solve the resulting equations for $\underline{\rho}$. Since this is such a well-known process, we go directly to the results without giving details. The equations to be solved for $\underline{\rho}$ can be expressed in terms of two matrices $\underline{\underline{A}}$ and $\underline{\underline{B}}$ having the dimensionless coefficients

$$\begin{aligned} A_{\alpha\beta} &= N^{-1} \sum_{\ell=1}^{N} \{v_{\perp\alpha}(t_\ell) v_{\perp\beta}(t_\ell)/n^2 R^2(t_\ell)\} \\ B_{\alpha\beta} &= N^{-1} \sum_{\ell=1}^{N} \{v_{\perp\alpha}(t_\ell)[t_\ell v_{\perp\beta}(t_\ell) + R_\beta(t_\ell)]/nR^2(t_\ell)\} \end{aligned} \tag{9}$$

In these, $v_{\perp\alpha}$, $v_{\perp\beta}$, and R_β refer to rectangular components of the vectors $\underline{v}_\perp$ and $\underline{R}$, n is the mean motion, and t_ℓ are the time points at which data are available. $\underline{\rho}$ satisfies the equation

$$\underline{\underline{A}}[(\underline{\rho} - \delta\underline{r}_i)/a] = \underline{\underline{B}}(\delta\dot{\underline{r}}_i/na) \quad ,$$

in which a is the semi-major axis. Finally, the solution for $\underline{\rho}$ is

$$(\underline{\rho}/a) = (\delta\underline{r}_i/a) + \underline{\underline{\sigma}}(\delta\dot{\underline{r}}_i/na) \qquad (10)$$

with $\underline{\underline{\sigma}} = \underline{\underline{A}}^{-1}\underline{\underline{B}}$.

It facilitates discussion to fix a particular coordinate system. Let the average orbit during a pass lie in the yz plane, and let the true station position lie in the xy plane. z is then the "along-track" coordinate. If the orbit is not too eccentric, only four coefficients of the $\underline{\underline{\sigma}}$ matrix are of appreciable size; these are σ_{yz}, σ_{xz}, σ_{zy}, and σ_{zx}.

The first two of these give position errors perpendicular to the trajectory resulting from the velocity error along the trajectory, the second two give position error along the trajectory resulting from the "cross-track" velocity errors $\delta\dot{\underline{r}}_{i,y}$ and $\delta\dot{\underline{r}}_{i,x}$. It is thus appropriate to combine the four coefficients into two,

$$\sigma_{C,L} = (\sigma_{yz}^{\ 2} + \sigma_{xz}^{\ 2})^{1/2} \quad , \qquad \sigma_{L,C} = (\sigma_{zy}^{\ 2} + \sigma_{zx}^{\ 2})^{1/2} \quad . \qquad (11)$$

In Table 1, we give some values of $\sigma_{C,L}$ and $\sigma_{L,C}$ calculated for several circular orbits of radius a, for each of three values of θ, the maximum elevation angle reached by the satellite during the pass. We have assumed that the data used cover 85% of the time that the satellite is above the horizon.

In Eq. (10), there is no necessary relation between $\delta\underline{r}_i$ and $\delta\dot{\underline{r}}_i$ for any one pass, but on the average the dimensionless errors $\delta\underline{r}_i/a$ and $\delta\dot{\underline{r}}_i/na$ should be of the same size, since a is the average radius and na is the average velocity. Then if the coefficients of $\underline{\underline{\sigma}}$ were less than unity, the difference between $\underline{\rho}/a$ and $\delta\underline{r}_i/a$ would be less in magnitude than $\delta\underline{r}_i/a$, and the substitution of $\underline{\rho}$ for $\delta\underline{r}_i$ would yield an improved estimate of the orbit.

From Table 1, we see that $\sigma_{L,C}$ is less than unity over the range of parameters studied. Therefore the "along-track" coodinate, the phase, is always improved by this process. $\sigma_{C,L}$, on the other hand, is greater than unity, and the "cross-track" coordinate tends to be degraded. If $\sigma_{C,L}$ were also less than unity, there would be no question about the convergence of the process; as matters stand, we are unable to give a rigorous theory of the convergence.

Our speculation is that the process is convergent if $\sigma_{L,C}$ is less than unity, almost regardless of the size of $\sigma_{C,L}$. Our reason is that the phase almost entirely controls the eccentricity and the position and time of perigee. Further, if the data cover a full revolution or more, the phase, because of the relation between a and the period, controls a more strongly than the altitude does. These four parameters control the altitude and the "along-track" velocity that will be computed from the new

Table 1

Effect of Errors in Satellite Velocity Upon Inferred Station Position

a Earth Radii	θ Degrees	$\sigma_{C,L}$	$\sigma_{L,C}$
1.2	15	2.15	0.354
	45	3.10	0.190
	75	8.58	0.145
1.4	15	1.84	0.490
	45	3.02	0.291
	75	8.64	0.226
1.6	15	1.26	0.563
	45	2.55	0.352
	75	7.82	0.278
1.8	15	2.13	0.607
	45	1.66	0.392
	75	5.94	0.315
2.0	15	5.26	0.636
	45	1.26	0.422
	75	2.88	0.342

orbit, thus we can expect that the new orbit will yield improved estimates of the cross-track position, in spite of the large values of $\sigma_{C,L}$. That is, we improve our estimates not because $\sigma_{C,L}$ is small, but because we decrease the quantity that $\sigma_{C,L}$ multiplies.

Of course, we cannot improve the orbit if $\sigma_{C,L}$ is too large. The criterion of convergence is probably one that depends upon the product of $\sigma_{C,L}$ and $\sigma_{L,C}$, and the critical value of the product is probably comparable to $3\pi/a$ times the number of revolutions spanned by the data. The reason for this guess is that an increment δa in a changes the altitude by just δa on the average, but changes the phase by $3\pi\delta a/a$ for each revolution.

The reason that $\sigma_{C,L}$ is so large is that we have tried to infer both altitude and horizontal cross-orbit position of the tracking station from the data. If the orbit were linear, this would be a singular process, and $\sigma_{C,L}$ would be infinite. For low orbits, the portion of the orbit used is nearly linear, and the process is nearly singular. As the value of a increases, the curvature of the orbit becomes more significant, and the process becomes well-conditioned. This probably accounts for the tendency of $\sigma_{C,L}$ in Table 1 to decrease with increasing a. This tendency is combatted by the tendency for the "fix" to become less precise as altitude increases, and at sufficiently large a, $\sigma_{C,L}$ might start to increase.

An Experimental Test

In the experimental testing we have done of the rapid orbit determination, we did not use exactly the method that has been described, because of the problem just mentioned. Instead of finding the station

position, in three dimensions, that best fits the orbit and the data, we find the best fit in two dimensions only. That is, in seeking the best fit, we constrain the inferred position of the station to lie on the horizontal plane through the nominal station position.

We can analyze the situation for this method of fitting by returning to Eq. (8) and allowing the vector $\underline{\rho}$ to have only horizontal components. The analysis is now more cumbersome because the symmetry of the former problem is lacking, and we shall only describe the nature of the results.

If we analyze $\underline{\rho}$ into components parallel and perpendicular to the satellite sub-track, the result for the parallel component is unchanged. That is, the values of $\sigma_{L,C}$ in Table 1 still apply. The sensitivity of the perpendicular component to error in the parallel velocity is considerably reduced, to about unity or less. However, this component of $\underline{\rho}$ no longer has any necessary relation to $\delta \underline{r}_1$. The horizontal component of $\delta \underline{r}_1$ normal to the average orbit plane shows up one-for-one in $\underline{\rho}$. The vertical component of $\delta \underline{r}_1$ also shows up in $\underline{\rho}$. An increase in altitude of the satellite moves the inferred station position toward the orbit plane. Overall, the magnitude of the difference between $\underline{\rho}$ and $\delta \underline{r}_1$, which is the main item of interest, is about the same whether $\underline{\rho}$ is allowed two components or three, while the calculations are faster if we allow only two.

We applied the modified method to the launching of ANNA 1B (1962 βμ1) on an experimental basis. We used the data from the first pass received after injection, the data from the pass one revolution later over the same station, and data from three other stations within

the same revolution. In Table 2, we compare the orbital elements obtained from this determination with those determined by the standard method using data for the first full 24 hours. The present method required only ten iterations; the standard method required about fifty.

The agreement between the two orbits is quite good, particularly in the most critical parameter a. The values for the time and argument of perigee differ considerably but this is an unimportant consequence of the low eccentricity of the orbit, since coordinates computed from the two sets of parameters differ by only about ten kilometers. Ten kilometers is not a serious error for this early stage in the satellite life: Just after launching, thermal and vacuum conditions are changing rapidly inside the satellite, hence, the satellite frequency is changing rapidly, and the Doppler data are not accurate.

In the calculations summarized in Table 2, we started both methods from the design orbit for the satellite. We later gave the present method a severe test by starting with an orbit known to be seriously in error. We chose a so as to make the period wrong by about two minutes, and chose the phasing to be so far off that the actual satellite was not above the horizon at the times that the starting orbit said it was. The present method still found a good estimate of the orbit in about fifteen iterations.

We have also tested the present method by taking an accurate orbit, determined after the satellite has stabilized, and finding the precessing ellipse that best fits the orbit. Since an ellipse is the orbital description used in the present method, this gives the best

Table 2

Comparison of Orbit Parameters Obtained From Doppler Data

Satellite: ANNA 1B (1962 βμ1)

Epoch: October 31.0, 1962

	Present Method	Standard Method
Semi-major axis	1.17717	1.17721
Eccentricity	0.00672	0.00670
Inclination	$50^{\circ}.12^{4}$	$50^{\circ}.18^{5}$
Longitude of node	$55^{\circ}.745$	$55^{\circ}.693$
Argument of perigee	$200^{\circ}.77$	$186^{\circ}.70$
Time of perigee	$6453^{s}.5$	$6198^{s}.5$
Approximate time span of data used	2 hours	24 hours

possible orbit that the present method could find. Applying the present method to the same data, we find no significant difference between the two elliptical orbits.

Extensions of the Method

With most iterative procedures, there is a region of convergence such that we converge to the solution starting from any point within the region. We can expect that this applies to the present computation process. If so, the process should continue to converge until it is blocked either by errors in the data or errors in the computation of an orbit from a given set of parameters. With this method as it exists at present, the orbital computation accuracy is by far the limiting one, because we have used a precessing ellipse as the orbital description. When we modify the orbit computation to agree with the computation used in the standard method, we can expect that the two methods will have the same accuracy.

The basic method is not limited to use with Doppler data. It can be used with any type of data that is capable of yielding a reasonable estimate of station position, given data from a single pass and an assumed orbit. So far as we can see, this applies to radio interferometric, radio ranging, or optical data, provided enough data points are obtained during a pass. An orbit computation process based on this method could then use any type or combination of types of data. One needs only a "station fixing" subroutine for each type of data. The output of the data from each pass, of whatever type, is then a station position error, which is assumed to represent the satellite position error at an epoch near the center of the

pass. All of the subroutine outputs then go to a single orbit determination routine, which takes the individual new estimates of satellite position, with weighting factors if desired, and finds the orbit that best fits these position estimates.

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